Functions

Derivation (part 2)

Important Notes

- If $f(x) = U \pm V$ then $f'(x) = U' \pm V'$
- If $f(x) = U.V$ then $f'(x) = U'V + V'U$
- If $f(x) = \frac{U}{V}$ then $f'(x) = \frac{U'V - V'U}{V^2}$

Derivative

Function	Derivative	Example
➢ $f(x) = k \quad k \in \mathbb{R}$	$f'(x) = 0$	$g(x) = -8 \; then \; g'(x) = 0$
➢ $f(x) = kx^n \quad k \,\&\, n \in \mathbb{R}$	$f'(x) = knx^{n-1}$	$h(x) = 5x^3 \; then \; h'(x) = 15x^2$
➢ $f(x) = kU^n$	$f'(x) = knU^{n-1}.U'$	$m(x) = 3(5x-2)^2$ $m'(x) = 3 \times 2(5x-2)^1 \times 5$
➢ $f(x) = k\sqrt{x}$	$f'(x) = \frac{k}{2\sqrt{x}}$	$u(x) = -5\sqrt{x}$ then $u'(x) = \frac{-5}{2\sqrt{x}}$
➢ $f(x) = k\sqrt{U}$	$f'(x) = \frac{k.U'}{2\sqrt{U}}$	$n(x) = 4\sqrt{3x^2-5x}$ $n'(x) = \frac{4.(6x-5)}{2\sqrt{3x^2-5x}}$

Solved Examples

$Ex1$

$$f(x) = 3(-4x^3 - 5x^2 + 1)^7$$

$$f'(x) = 21\left(-4x^3 - 5x^2 + 1\right)^6(-12x^2 - 10x)$$

Solved Examples

Ex2

$$f(x) = \frac{5}{x} - \frac{3}{x^2} + \frac{2}{x^4}$$

$$x^{-n} = \frac{1}{x^n}$$

$$f(x) = 5x^{-1} - 3x^{-2} + 2x^{-4}$$

$$f'(x) = -5x^{-2} + 6x^{-3} - 8x^{-5}$$
$$= \frac{-5}{x^2} + \frac{6}{x^3} - \frac{8}{x^5}$$

Solved Examples

Ex3

$$f(x) = \sqrt{7x^2 + 5x + 1} + \frac{3}{\sqrt{x}}$$

$$f(x) = \sqrt{7x^2 + 5x + 1} + 3x^{\frac{-1}{2}}$$

$$\sqrt{x} = x^{\frac{1}{2}}$$

$$f'(x) = \frac{14x+5}{2\sqrt{7x^2+5x+1}} - \frac{3}{2}x^{\frac{-3}{2}}$$

$$= \frac{14x+5}{2\sqrt{7x^2+5x+1}} - \frac{3}{2x\sqrt{x}}$$

$$x^{\frac{3}{2}} = \sqrt{x^3} = x\sqrt{x}$$

Solved Examples

$$f(x) = UV$$
$$f'(x) = U'V + V'U$$

Ex4

$$f(x) = (2x+1)(3x^2-5x)^4$$

$$U = 2x+1$$
$$V = (3x^2-5x)^4$$

$$U' = 2$$
$$V' = 4(3x^2-5x)^3(6x-5)$$

$$f'(x) = 2(3x^2-5x)^4 + (2x+1)4(3x^2-5x)^3(6x-5)$$

Solved Examples

$$f(x) = \frac{U}{V}$$

$$f'(x) = \frac{U'V - V'U}{V^2}$$

Ex5

$$f(x) = \frac{2x+1}{(3x^2-5x)^4}$$

$$U = 2x + 1$$
$$V = (3x^2 - 5x)^4$$

$$U' = 2$$
$$V' = 4(3x^2 - 5x)^3(6x - 5)$$

$$f'(x) = \frac{2(3x^2-5x)^4 - (2x+1)4(3x^2-5x)^3(6x-5)}{[(3x^2-5x)^4]^2}$$